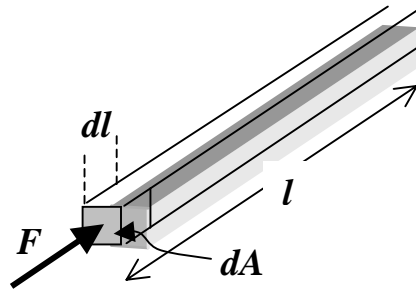


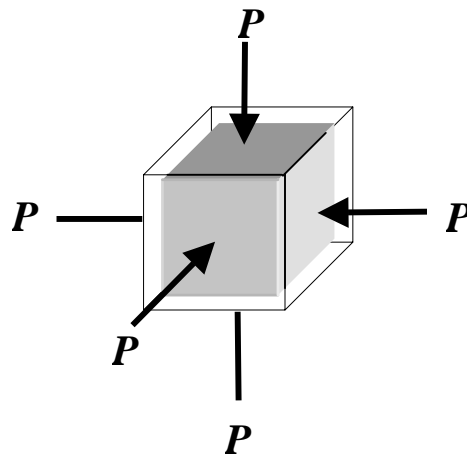
## Stress, Strain and Hooke's Law

In very early studies of mechanical deformation under applied stress Hooke found that the deformation was linearly proportional to the stress *for small deformations*. The simplest of these observations was made by Young for extensions or contractions of a thin rod under axial load. For a model of a rod with rectangular cross section of area  $dA$  and length  $l$  the ratio of the stress, the applied axial force divided by the cross sectional area, to the strain, the change in length  $dl$  divided by the length is a constant. The constant now known as Young's modulus, is given by:

$$\text{Young's modulus, } E = \frac{\text{stress}}{\text{strain}} = \frac{\frac{F}{dA}}{\frac{dl}{l}}$$



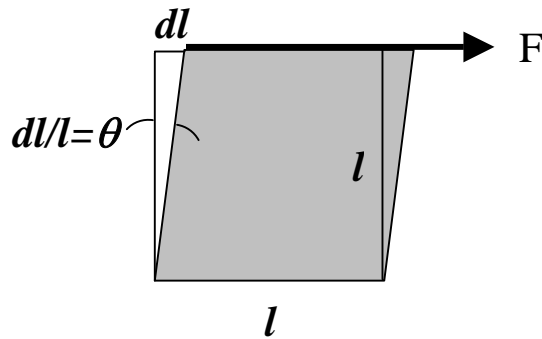
The uniform compression of a fluid or solid under hydrostatic pressure (again force divided by area) produces a strain which is the fractional change in volume and the constant of proportionality is the bulk modulus  $K$ . For the following cube:



$$\text{Bulk modulus, } K = \frac{P}{\frac{dV}{V}}$$

Finally for an element in shear, the force is applied tangentially to one of the sides of the cube (the other is assumed glued to an immovable plane in the following cartoon). The stress is now the force divided by the area over which the shear is applied, and the strain is the lateral deformation  $dl$  divided by height of the element,  $l$ . For small deformations the latter ratio is simply the angular deformation  $\theta$

$$\text{Shear modulus, } \mu = \frac{F/l^2}{\theta}$$



Collectively, these stress strain relationships are known as Hooke's Law.

One more important physical property is Poisson's relationship. When an object is compressed (or extended) it is observed that it expands (or contracts) perpendicular to the direction of the applied stress. This is illustrated by the compression of the rectangular block in the following cartoon. The ratio of the transverse strain to the axial strain is Poisson's ratio.

Poisson's ratio,  $\sigma = \frac{\frac{dr}{r}}{\frac{dl}{l}}$

